Complex number 3

1. Given that $z+\frac{1}{z}=1$, find the values of **(a)** $z^4+\frac{1}{z^4}$ **(b)** $z^5+\frac{1}{z^5}$.

2. **(a)** Using deMoivre's Theorem to show that $\sin 5\theta = a\, sin^5\,\theta + b cos^2\,\theta sin^3\,\theta + cos^4\,\theta\, sin\,\theta$, where a, b and c are integers to be determined.

(b) Express $\frac{\sin 5\theta}{sin\,\theta}$ in terms of $\cos\theta$, where θ is not a multiple of π.

Hence, find the roots of the equation $16x^4 - 12x^2 + 1 = 0$ in trigonometric form.

3. **(a)** Find the roots of $z^5 = 1$.

(b) Show that one of the roots in **(a)** is $\omega = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5} = \frac{\sqrt{5}-1}{4} + i\frac{\sqrt{10+2\sqrt{5}}}{4}$

(c) Show that **(i)** $1+\omega+\omega^2+\omega^3+\omega^4 = 0$,

(ii) $|1+\omega^2+\omega^4| = \sqrt{\frac{3-\sqrt{5}}{2}}$.

4. Show that $-\sqrt{2}+i\sqrt{2}$ is a root if $x^4+16=0$. The root $-\sqrt{2}+i\sqrt{2}$ is located on a circle of radius 2 in an Argand diagram and plot all the roots.

5. Solve the equation $5z^4 - z^3 + 4z^2 - z + 5 = 0$.

6. ABCD is a square with the letters in the anticlockwise order. The points A and B represents $2+3i$ and $6+i$ respectively. Find the complex number represented by C and D.

7. The equation $z^4 - 2z^3 + kz^2 - 18z + 45 = 0$ has imaginary roots. Obtain all the roots of the equation and the value of the real constant k.

8. **(a)** Let $z = -2-3i$, find z^2.

(b) Hence solve the equations : (i) $w^2 + 4w = -9 + 12i$

(ii) $w^4 + 4w^2 = -9 + 12i$.

9. If $z = \cos\theta + i\sin\theta$, show that $\frac{1}{1+z^2} = \frac{1}{2}(1 - i\,tan\,\theta)$ and write $\frac{1}{1-z^2}$ in similar form.

10. **(a)** Let $z = 1+i$, find $z^n, n = 1,2,3,4,\ldots$ in Cartesian form.

(b) Find z^n in polar form.

(c) Explain, without plotting, how you can represent z^n in Argand diagram.